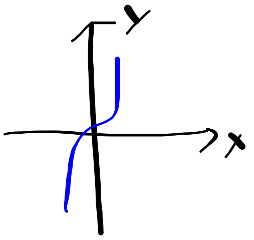
2.3 Higher Degree Polynomial Functions

Graphing combinations of Monomials

Ex.1 Graph $h(x)=x^3+x$. How does it relate to the two monomials from which it was built? Locate its extrema and zeros?



$\lim_{x\to\infty} h(x) \to \infty$

$\lim_{x\to -\infty} h(x) \to -\infty$

$\lim_{x\to 0^+} h(x) \to x$

$\lim_{x\to 0^-} h(x) \to x$

Repeat for $g(x)=x^3-x$.

Polynomials are smooth continuous functions with no sharp corners or cusps.

A polynomial of degree **n** has at most **n-1** local extrema and at most **n** zeros.

$f(x) = 5x^{27} - 3x^9 + 2x^2 - 4$

degree $= 27$

# of possible local extrema $= 26$

# of possible zeros $= 27$

Odd functions have opposite left and right end behaviors and at least one zero.

Even functions have the same left and right end behaviors.

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Finding Zeros of a Polynomial Function

Ex.2 Find the zeros of $f(x) = x^3 + 9x^2 + 8x$ do this algebraically and support graphically.

$$x^3 + 9x^2 + 8x = 0$$

$$x(x^2 + 9x + 8) = 0$$

$$x(x+8)(x+1) = 0$$

zeros $x = -8, -1, 0$

TRY Repeat above for $g(x) = -(x+4)^3(x-7)^9(x+3)^4(x-12)^1$

degree $3+9+4+1 = 17$

# of possible local extrema 16

# of possible zeros 17

What the actual zeros are $-4, -3, 7, 12$

$$\lim_{x \to \infty} g(x) \to -x^{17} = -\infty$$

$$\lim_{x \to -\infty} g(x) \to -x^{17} = \infty$$

The degree of a function is the degree of the highest exponent.

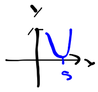
After a function is fully factored it is easy to see the multiplicity of a function. Multiplicity is the degree of each factor.

$f(x) = (x+4)^3(x-3)^7(x-5)^{12}$

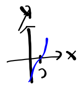
zeros $x = -4 \quad 3 \quad 5$

Multiplicity $3 \quad 7 \quad 12$

Even multiplicity zeros kiss the axis.



Odd multiplicity zeros cross the axis.



Ex.5

State the degree and list the zeros of the function

$$f(x) = x(x+2)(x+1)^3$$

State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Sketch the graph BY HAND.

degree 5

poss. local extrema 4

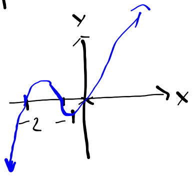
poss # of zeros 5

Actual zeros $-2 \quad -1 \quad 0$ $\qquad 0 \quad -2 \quad -1$

multiplicity $1 \quad 3 \quad 1$ $\qquad 1 \quad 1 \quad 3$

$\lim_{x\to\infty} f(x) \to x^5 = \infty$

$\lim_{x\to-\infty} f(x) \to x^5 = -\infty$



TRY Repeat example 5 for $g(x) = x^2(x+2)(x+1)^3$

TRY Repeat above for $g(x) = -(x+4)^3(x-7)^9(x+3)^4(x-12)^6$

degree 22

\# poss. LE 21

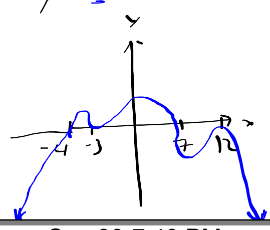
\# poss. zeros 22

Actual zeros $-4 \quad -3 \quad 7 \quad 12$

Multiplicity $3 \quad 4 \quad 9 \quad 6$

$\lim_{x\to\infty} g(x) = -\infty$

$\lim_{x\to-\infty} g(x) = -\infty$



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Ex. 3

Write a cubic polynomial for the function with zeros -1, -2 and 3.

$$f(x) = (x-3)(x+1)(x+2)$$

FOIL

$$= (x-3)(x^2+3x+2)$$

$$x^3+3x^2+2x$$
$$-3x^2-9x-6$$

$$f(x) = x^3 - 7x - 6$$

TRY

Write a cubic polynomial for the function with zeros 1, 1±√5 .

$$f(x) = (x-1)(x-(1+\sqrt{5}))(x-(1-\sqrt{5}))$$

$$= (x-1)(x^2-(1-\sqrt{5})x-(1+\sqrt{5})x+(1-\sqrt{5})(1+\sqrt{5}))$$

$$=(x-1)(x^2-x+\cancel{\sqrt{5}x}-x-\cancel{\sqrt{5}x}+1+\cancel{\sqrt{5}}-\cancel{\sqrt{5}}-5)$$

$$=(x-1)(x^2-2x-4)$$

$$= x^3-2x^2-4x$$
$$-x^2+2x+4$$

$$f(x) = x^3-3x^2-2x+4$$